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DEFINITION OF THE INFLUENCE VECTOR IN EARTHQUAKE ANALYSIS

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ABSTRACT

Procedures for computing the response of structures during an earthquake (or another dynamic ground motion) involves a calculation of the modal participation factor, which formula includes influence vector *{r}*. The definition of the influence vector is often given as a "displacement transformation vector that expresses the displacement of each structure degree of freedom due to static application of a unit support displacement" or something similar. That kind of definitions produces correct results only in regular structures depending on the directions of the DOFs and the ground excitation. But in many cases the usage the definitions mentioned above leads to wrong results, even sometimes to zero solutions. In this paper another method to calculate the modal participation factor is proposed based on the usage of the load distribution vector *{R}* and it is correct in all cases. A correct formula for the influence vector as a function of load distribution vector is also proposed.

1. Introduction

The simplest form of earthquake response problem involves a SDOF lumped mass system subjected to identical single component translations of all support points.

$$m\ddot{u}(t)+c\dot{u}(t)+k\,u(t)=p_{eff}(t), \qquad (1)$$

where $p_{eff}(t)=-m\ddot{u}_g(t)$. In the last expression $\ddot{u}_g(t)$ is the free-field input acceleration applied at the base of the structure.

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The formulation of the earthquake response analysis of a lumped MDOF system can be carried out in matrix notation in a manner entirely analogous to the foregoing development of the lumped SDOF equations.

$$[m]\left\{\ddot{u}(t)\right\}+[c]\left\{\dot{u}(t)\right\}+[k]\left\{u(t)\right\}=\left\{p_{eff}(t)\right\}, \tag{2}$$

where $\left\{p_{eff}(t)\right\}=[m]\{r\}\ddot{u}_g(t)$. In [2] the vector $\{r\}$ is introduced as a "displacement transformation vector that expresses the displacement of each structure degree of freedom due to static application of a unit support displacement". Similarly the same vector (noted with symbol ι) is described as an "influence vector represents the displacement of the masses resulting from static application of a unit ground displacement" in [3].

Those statements are not valid for all cases of assumed degrees of freedom and types of earthquake excitation. The following example will show why.

2. Inclined cantilever example

In the given simple example, the structure has two degrees of freedom as we assume that axial deformations are negligible.

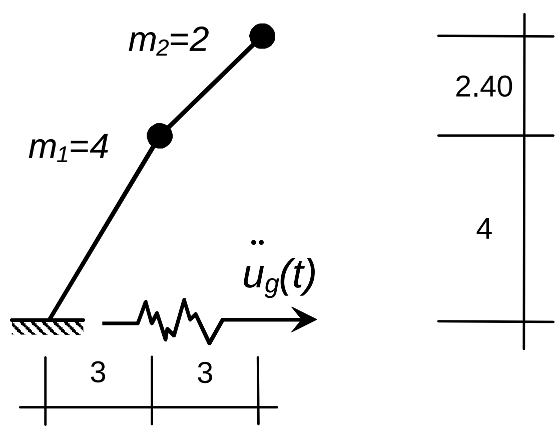

Figure 1. Cantilever with two DOFs, subjected on an horizontal ground excitation

The natural frequencies are $\omega_1 = 7.070992$ and $\omega_2 = 37.4426$, the natural periods are $T_1 = 0.888586$ and $T_2 = 0.167809$ [s.] and the values from elastic response spectra $S_a(T_n)$ are 1.35 and 2.50 . The corresponding normalized eigen modes are scheduled on the Fig. 2.,

Herewith three variants of the adopted DOFs are considered depending on the convenience we are looking for. In all of them the modal masses $M_n = \{\phi_n\}^T[m]\{\phi_n\}$ are equal to unity and therefore the modal participating factor MPF_n ,(often noted as Γ_n [1]), which shows how an eigen mode n participates in the earthquake response $MPF_n = L_n/M_n = L_n$, where $L_n = \{\phi_n\}^T \{R\}$ is the modal earthquake excitation factor. In our example $MPF_1 = 1.66142$, $MPF_2 = 1.07008$.

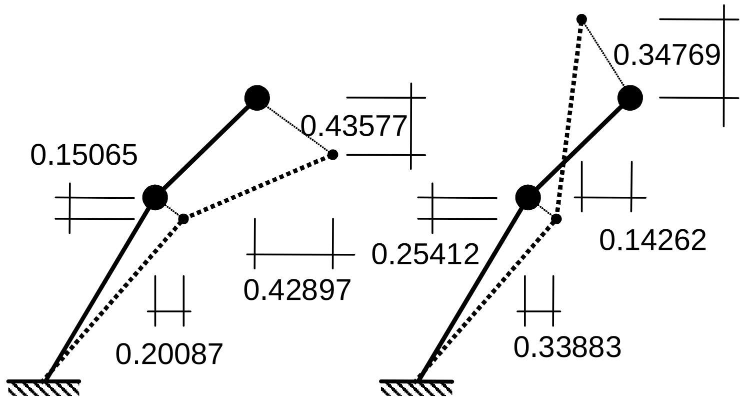

Figure 2. Normalized eigen modes

The relationship between vectors is $\{R\} = [m]\{r\}$ and it can be used aiming to express the influence vector $\{r\}$ as:

$$\{r\} = [m]^{-1}\{R\}, \tag{3}$$

Actually the acceleration loads that act on the structure due to unit acceleration in the direction of the ground motion should be computed. They are determined by d'Alembert's principle of dynamic equilibrium. Acceleration loads are simply equal to the negative of the lumped translational masses in case of unit translational ground acceleration. The elements of the load distribution vector $\{R\}$ are the reactions in the links along the dynamics DOFs due to acceleration loads.

The maximum elastic-force vector in mode n is given by

$$\{f_{Sn,\max}\} = [m]\{\phi_n\} \frac{L_n}{M_n} S_a(T_n), \tag{4}$$

where $S_a(\xi_n, T_n)$ is the spectral acceleration for the n^{-th} mode [2]. In the example the design force vector $\{E_n\}$ in mode n is given by $\{E_n\} = 0.0675 \times \{f_{Sn,\max}\}$, where the (seismic) coefficient 0.0675 includes effective peak acceleration coefficient (mapped spectral acceleration in % of g=9.81$\left[m/s^2\right]$), importance factor (=1) and response modification factor according to the old Bulgarian Code for Design[4].

2. Comparison of the three variants

All variants lead to the same natural frequencies and modes, same action effects on the structure - displacements and internal forces. The ways to obtain those results are different.

2.1. Diagonal

Degrees of freedom are assumed to be orthogonal to the elements' axis. The advantage of the "diagonal" variant is that a mass matrix is diagonal. Its elements should be found by applying an unit acceleration consecutively along every DOF, keeping all other accelerations to zero.

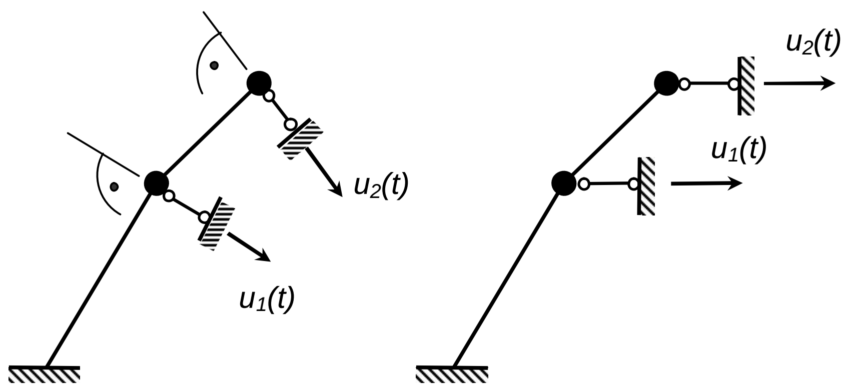

Figure 3. Assumed degrees of freedom in the "diagonal" and "horizontal" variants

The mass influence element $m_{11} \neq m_1$, because of the geometry of the system (elements have different slope). The other mass influence element $m_{22} = m_2$. The values of $\{r\}$ can't be found as a trigonometric function of any angle between directions of ground motion and degrees of freedom.

Table 1. Calculations based on the "Diagonal" variant

Variant	Diagonal
$[m], [k]$	$\begin{bmatrix} 4.124878 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 4332.61 & -1703.39 \\ -1703.39 & 803.167 \end{bmatrix}$
$\{\phi_n\}$	$\{\phi_1\} = \begin{Bmatrix} 0.25109 \\ 0.60825 \end{Bmatrix}; \{\phi_2\} = \begin{Bmatrix} 0.42354 \\ -.36060 \end{Bmatrix}$
$\{R\}^T$	$\{3.59024 \quad 1.24939\}$
$\{r\}^T$	$\{0.88039 \quad 0.62469\}$
$\{E_n\}$	$\{E_1\} = \begin{Bmatrix} 1.5383 \\ 1.8068 \end{Bmatrix}; \{E_2\} = \begin{Bmatrix} 3.0949 \\ -1.2776 \end{Bmatrix}$

2.2. Horizontal

The advantage of the "horizontal" variant is that obtaining the elements of the load distribution vector $\{R\}$ is easy. They are simply equal the mass values.

The influence vector $\{r\} \neq \{1 \quad 1\}^T$ and can be calculated by the proposed formula (3):

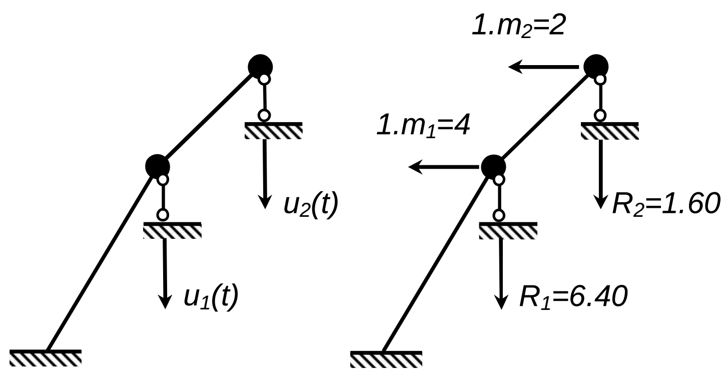

Figure 4. (a) Degrees of freedom in the "vertical" variant; (b) Applied horizontal inertial forces due to unit ground acceleration and received vertical reactions

2.3. Vertical

The advantage of the "vertical" variant is that it is absolutely obvious that applying literally the sentence that the influence vector $\{r\}$ "expresses the displacement of each structure degree of freedom due to static application of a unit support displacement" [2] results in a zero solution, which is absurd.

On Fig. 4 are shown the forces due to unit acceleration of the ground in the given direction – horizontal.

Table 2. Calculations based on the "horizontal" and "vertical" variants

Variant	Horizontal	Vertical
$[m]$	$\begin{bmatrix} 6.75 & -1.25 \\ -1.25 & 5.125 \end{bmatrix}$	$\begin{bmatrix} 11.68 & 0.853333 \\ 0.853333 & 3.28 \end{bmatrix}$
$\{\phi_1\}^T$	$\{0.20087 \quad 0.42897\}$	$\{0.15065 \quad 0.43577\}$
$\{\phi_2\}^T$	$\{0.33883 \quad -.14262\}$	$\{0.25412 \quad -.34769\}$
$\{R\}^T$	$\{4 \quad 2\}$	$\{6.4000 \quad 1.6000\}$
$\{r\}^T$	$\{0.69631 \quad 0.56008\}$	$\{0.52223 \quad 0.35194\}$
$\{E_1\}^T$	$\{1.2174 \quad 2.8923\}$	$\{3.1658 \quad 2.3139\}$
$\{E_2\}^T$	$\{4.3675 \quad -2.0452\}$	$\{4.7326 \quad -1.6361\}$

Applying the design force vector on the structure along assumed directions (different for every variant) of the DOFs gives the internal forces for the n^{-th} mode. Using the SRSS method to these modal results gives the maximum response bending moment, which values are the same in every variant: $M_{base} = 23.79\ [kNm]$ and $M_{h4} = 8.50\ [kNm]$.

Table 3. Bending moments

section	case	Bending moment [kN.m]
base	Mode 1/ Mode 2	23.38 / 4.38
	SRSS	**23.79**
h=4 (at the middle)	Mode 1/ Mode 2	6.94 / -4.91
	SRSS	**8.50**

4. Conclusion

The modal earthquake excitation factor L_n often is expressed as a function of $\{r\}$, together with incorrect definition, because many authors express $\{r\}$ as a displacement, which is a function of some unit displacement. Actually it should be found the load distribution vector $\{R\}$ as a function of unit acceleration. After that it can be found $\{r\}$ by (3) or better directly the modal earthquake excitation factor $L_n = \{\phi_n\}^T \{R\}$.

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